# 分形几何及相关问题

**时间：2022.10.15-10.17**

**地点：腾讯会议：488-9707-7423 会议密码：221015**

**邀请人：武文，熊瑛**

**报告日程安排**

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| --- | --- | --- | --- |
| 10月15日（周六） | | | |
| 时间 | | **报告人** | **报告题目** |
| 上午 | 9:00-9:40 | 文志雄 | Hankel determinant, Pade approximant and irrationality exponent |
| 9:45-10:25 | 林勇 | Normalized discrete Ricci flow and community detection |
| 10:35-11:15 | 李文侠 | How inhomogeneous Cantor sets can pass a point |
| 11:20-12:00 | 孔德荣 | Univoque bases of real numbers: simply normal bases, irregular bases and multiple rationals |
| 下午 | 14:30-15:10 | 董新汉 | Fractal Hardy spaces |
| 15:15-15:55 | 邱华 | Dirichlet forms on unconstrained Sierpinski carpets |
| 16:05-16:45 | 廖灵敏 | Normal numbers in base 2 with given limit of multiple ergodic averages |
| 16:50-17:30 | 刘竟成 | The spectrality of planar self-affine measures with four digits |
| 10月16日（周日） | | | |
| 时间 | | **报告人** | **报告题目** |
| 上午 | 9:00-9:40 | 丰德军 | Typical self-affine sets with non-empty interior |
| 9:45-10:25 | 倪思敏 | Laplacians defined by fractal measures on Riemannian manifolds |
| 10:35-11:15 | 马际华 | A Billingsley type theorem for the Bowen entropy |
| 11:20-12:00 | 刘庆晖 | Spectrums of substitutional Hamiltonians |
| 10月17日（周一） | | | |
| 时间 | | **报告人** | **报告题目** |
| 下午 | 14:30-15:10 | 文志英 | 分形几何中的一些问题 |
| 15:15-15:55 | 饶辉 | Box-counting measure of metric space |
| 16:05-16:45 | 王保伟 | Measure theoretic laws for limsup sets of rectangles in Diophantine approximation |

## 报告信息

**报 告 人**：文志雄 教授（华中科技大学）

**报告题目**：Hankel determinant, Padé approximant and irrationality exponent

**报告摘要**：In this talk, we will introduce some methods for calculating Hankel determinants of automatic sequences, and give applications in the study of the Diophantine properties of automatic numbers --- real numbers whose b-array expansions are automatic sequences. In particular, we will discuss a class of automatic sequences, called the apwenian sequences, whose Hankel determinants (mod 2) coincide with those of the well-known Thue-Morse sequence. Some criteria for telling if automatic sequences on {+1, -1} and {0, 1} are apwenian or not will be introduced.

**报 告 人**：林勇 教授（清华大学）

**报告题目**：Normalized discrete Ricci flow and community detection

**报告摘要**：我们证明了由图上离散的Ricci曲率定义的曲率流方程解的存在唯一性。同时我们利用这种曲率流方程研究图分割问题，并且比较了我们的图分割方法和其他经典的图分割方法。

**报 告 人**：李文侠 教授（华东师范大学）

**报告题目**：How inhomogeneous Cantor sets can pass a point

**报告摘要**：设为两参数康托集，由迭代函数系统所确定. 固定正数，记我们证明集合的勒贝格测度为零. 但Hausdorff维数为2. 此外，对于任意有限个正数，我们证明集合的交集为Hausdorff维数仍然为2.

**报 告 人**：孔德荣 教授（重庆大学）

**报告题目**：Univoque bases of real numbers: simply normal bases, irregular bases and multiple rationals

**报告摘要**：Given a positive integer and a real number in , we show that the set of bases in which the unique -expansion of is simply normal has full Hausdorff dimension. Moreover, we show that the set of in which the unique -expansion of has no digit frequency also has full Hausdorff dimension. Finally, given finitely many rationals in so that each has a finite expansion in base , we show that the set of in which each has a unique -expansion has full Hausdorff dimension. This is joint work with Yu Hu and Yan Huang.

**报 告 人**：董新汉 教授（湖南师范大学）

**报告题目**：Fractal Hardy spaces

**报告摘要**：The first singular, non-atomic, spectral measure was constructed by Jorgensen and Pedersen, they proved that the -Cantor measure on is a spectral measure, and is a spectrum of , where For , we let where . Obviously, is analytic in , and , hence is a complete subspace of (Hardy space). We call a fractal Hardy spaces. From Fatou Theorem, the non-tangential limit of exists almost everywhere on , and equals to the radial limit. That is, there is an exceptional set (we say Fatou's exceptional set)   
such that and exist for . It is easy to see that the trigonometric series is exactly right the Fourier series of , in the sense of .  
We mainly investigated the following problems: ♣ Integral representation theorem of ; ♣ Growth theorem of ; ♣ Strichartz's exceptional set and Carleson's exceptional set under certain conditions; ♣ Picard property, Cantor boundary behaviour, of , under Hadamard gap condition. The following important function spaces and theorems are involved: Dirichlet space , Lipschitz class of order , Abel's theorem, Littlewood's Tauberian's theorem, Fejer's Tauberian's theorem, Bernstein's theorem and Hardy-Littlewood theorem.

**报 告 人**：邱华 教授（南京大学）

**报告题目**：Dirichlet forms on unconstrained Sierpinski carpets

**报告摘要**：In this talk, I will talk about how to construct self-similar Dirichlet forms on un-constrained Sierpinski carpets (USC) in a purely analytic way. Comparing to the Sierpinski carpets (SC), the USC are more flexible in geometry as cells except those along the boundary are allowed to live off the grids. In 90’s, there are two probabilistic approaches to the existence of self-similar Dirichlet forms on SC due to Barlow-Bass and Kusuoka-Zhou respectively, which are shown to be equivalent in 2010. But the analytic approach remains unknown. This talk is based on a joint work with Shiping Cao.

**报 告 人**：廖灵敏 教授（武汉大学）

**报告题目**：Normal numbers in base 2 with given limit of multiple ergodic averages

**报告摘要**：It is shown that the Hausdorff dimension of the set of normal numbers in base 2 whose binary expansions have the property that for any k, the k-th digits and the 2k-th digits are not all equal to 1, is one-half. Further, the Hausdorff dimension formula for the set of normal numbers in base 2 with given limit of multiple ergodic averages of some special function is determined. This is a joint work with Michal Rams.

**报 告 人**：刘竟成 教授（湖南师范大学）

**报告题目**：The spectrality of planar self-affine measures with four digits

**报告摘要**：Let be the planar self-affine measure generated by an expansive integer matrix and a non-collinear integer digit set   
In this talk, I will show that the self-affine measure is a spectral measure if and only if there exists a matrix such that is admissible, where and . This is a joint work with Mingliang Chen.

**报 告 人**：丰德军 教授（香港中文大学）

**报告题目**：Typical self-affine sets with non-empty interior

**报告摘要**：In this talk, I will present some sufficient conditions for a typical self-affine set to have non-empty interior. This is based on joint work with Zhou Feng.

**报 告 人**：倪思敏 教授（湖南师范大学，佐治亚南方大学）

**报告题目**：Laplacians defined by fractal measures on Riemannian manifolds

**报告摘要**：For a bounded open set in a complete -dimensional oriented Riemannian manifold and a positive finite Borel measure µ with support contained in the closure of , we define an associated Dirichlet Laplacian by assuming the Poincaré inequality. We obtain sufficient conditions for the Laplacian to have compact resolvent and in this case, we prove the Hodge theorem for functions, which states that there exists an orthonormal basis of consisting of eigenfunctions of the Laplacian, the eigenspaces are finite-dimensional, and the eigenvalues are real, countable, and increasing to infinity. The main tool we use is the triangle comparison theorem. One of these sufficient conditions is that the -dimension of is greater than . We study this condition for self-similar and self-conformal measures. Results in this paper extend analogous ones by Hu, Lau, and Ngai. in [J. Funct. Anal. 239 (2006), 542–565]. This work is joint with Lei Ouyang.

**报 告 人**：马际华 教授（武汉大学）

**报告题目**：A Billingsley type theorem for the Bowen entropy

**报告摘要**：For subsets of a metric space with a continuous map, Bowen introduced a notion of entropy. We show that the Bowen entropy can be determined via the local entropy of measures. This result can be regarded as an analogue of the Billingsley theorem for the Hausdorff dimension.

**报 告 人**：刘庆晖 教授（北京理工大学）

**报告题目**：Spectrums of substitutional Hamiltonians

**报告摘要**：We introduce new results on spectrum of 1-dim Schrodinger operator with poten-tials generated by periodic doubling substitution and generalized Thue-Morse substitutions.

**报 告 人**：文志英 教授（清华大学）

**报告题目**：分形几何中的一些问题

**报告摘要**：介绍分形几何中大家关心的一些问题，这些问题的历史、现状以及对分形几何可能产生的影响。

**报 告 人**：饶辉 教授（华中师范大学）

**报告题目**：Box-counting measures of metric spaces

**报告摘要**：We introduce a new measure, called box-counting measure for metric spaces. We prove that box-counting measure is ‘invariant’ under a bi-Lipschitz map. Finally, we show that several classes of self-affine sponges admit box-counting measure.

**报 告 人**：王保伟 教授（华中科技大学）

**报告题目**：Measure theoretic laws for limsup sets of rectangles in Diophantine approximation

**报告摘要**：We will talk about the measure theory for limsup sets generated by rectangles in Diophantine approximation including the general principles for Lebesgue measure theory and Hausdorff measure theory.