Low-Complexity Resource Allocation Algorithm for Multicell OFDMA System

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SUMMARY Low-complexity joint subcarrier and power allocation is considered. The applied criterion is to minimize the transmission power while satisfying the users’ rate requirements. Subcarrier and power allocation are separately applied. Fixed spectrum efficiency is assumed to simplify the subcarrier allocation. We show that under fixed spectrum efficiency, power allocation can be obtained by solving some sets of linear equations. Simulation result shows the effectiveness of the proposed algorithm.

key words: Resource Allocation; OFDMA; Cellular System

1. Introduction

Orthogonal frequency division multiple access (OFDMA) has become the major multiple access scheme for broadband wireless systems including IEEE 802.16, IEEE 802.20 and 3GPP long term evolution (LTE). The performance of OFDMA transmission systems can be optimized through three main mechanisms [1-3]: (1) subcarrier assignment, (2) bit loading, and (3) power loading. It has been shown that obtaining the optimal solution jointly is typical NP-hard[3-4]. In cellular OFDMA systems, resource allocation should consider the intercell interference (ICI), which makes the resource allocation extremely complicated [1-4].

Sub-optimal approaches with low complexity have attracted remarkable research interest. By some practical assumptions, the NP problem can be approximated as a linear programming problem, which significantly reduces the complexity [5-6]. Heuristic algorithms have also been proposed to reduce the complexity [7-9]. Another important technique is to apply distributed algorithm, where each cell by itself decides the resource allocation. User information, such as channel state, location is exchanged between neighboring basesations to coordinate intercell interference [4,10].

In this paper, we propose a low-complexity joint subcarrier and power allocation algorithm for OFDMA systems. The optimization target is minimizing the transmission power under the constraint that each user has a rate requirement. We assume each user has a fixed spectral efficiency in its assigned subcarriers. User’s rate requirement then is converted to a required number of subcarriers, which can simplify the subcarrier allocation. We further show that under the assumption of fixed spectrum efficiency, power allocation can be obtained by solving some sets of linear equations. Adjustment of power allocation is also proposed in case that the linear equations have negative solutions.

2. System Model

We consider the downlink of multicell OFDMA systems. The problem in the multicell scenario is formally described in the following. We are given a set of subcarriers \( M \), a set of cells \( C \), and for each cell \( k \in C \) a set of users \( U_k \). Let \( U = \bigcup U_k \) denotes the set of all users. For each user \( i \in U \), we denote by \( b(i) \) the serving cell of user \( i \). Hence, \( b(i) = k \) for all \( i \in U_k \).

For simplicity, we apply the assumption in [4,9,10], which set for each user a certain target spectral efficiency \( \eta_i \) (in bits/Hz). Hence, transmission requirements for a given user \( i \) correspond to a certain number of subcarrier \( r_i = R_i / B \eta_i \), where \( R_i \) is the transmission rate required by user \( i \). \( B \) is the bandwidth of each subcarrier and \( \eta_i \) is set in such a way that \( r_i \) is an integer. We note that the target spectral efficiency can be further converted to a requirement of signal-to-noise-and-interference-ratio (SINR). Assume Gaussian white channel noise. According to Shannon’s law \( \eta_i = \log_2(1 + SINR_i) \), the required SINR for user \( i \) in the assigned subcarrier is \( SINR_i = 2^{\eta_i} - 1 \).

In general, users belonging to different cells can share the same subcarrier. However, the power to be transmitted on a given subcarrier increases as the number of users transmitting on that subcarrier increases. More precisely, let \( S(j) \subseteq U \) be the set of users (belong to different cells) which are assigned the same subcarrier \( j \). The measured SINR for user \( i \) on subcarrier \( j \) is

\[
SINR_i(j) = \frac{p_i(j)G_i(j)}{\sum_{k \in S(j), k \neq i} C_{ki}G_{ki}(j)p_k(j) + BN_0}, \quad i \in S(j)
\]  

where \( p_i(j) \) is the transmission power allocated to user \( i \) on sub-carrier \( j \), \( G_i(j) \) is the channel power gain of user \( i \) on sub-carrier \( j \) (i.e. the channel power gain between user \( i \) and its serving basestation), \( G_{ki}(j) \) is the channel power gain between user \( i \) and the basestation of cell \( k \neq b(i) \) on subcarrier \( j \). \( G_{ki}(j) \) is a measure of the interference between user \( i \) and users of other cells transmitting on the same subcarrier \( j \).
We assume $G_i^h(j)$ are measured by the basestation and shared through the interface of basestations.

Note that $\text{SINR}_i$ corresponding to the spectral efficiency $\eta_i$. To fulfill the rate requirement of all users, it is required that $\text{SINR}_i(j) \geq \text{SINR}_i$ for power allocation. Based on this, we can formulate the joint subcarrier-power allocation problem as follows.

$$\min \sum_{i \in i,j,M} x_i(j)p_i(j) \quad (2)$$

$$\text{St} \quad p_i(j) \geq \text{SINR}_i \frac{\sum_{j \in i,j,M} G_i^h(j)p_i(j) + BN_0}{G_i(j)}, i \in S(j) \quad (3)$$

$$\sum_{i \in i,j,M} x_i(j)p_i(j) \leq p_{\text{max},k}, \quad p_i(j) \geq 0 \quad (4)$$

$$\sum_{i \in i,j} x_i(j) = r_j, \quad \sum_{i \in i,j} x_i(j) \leq 1 \quad (5)$$

where $x_i(j) = \{0,1\}$ is the indicator of subcarrier allocation. If subcarrier $j$ is assigned to user $i$ then $x_i(j) = 1$, otherwise $x_i(j) = 0$. Constraint (3) is to guarantee $\text{SINR}_i(j) \geq \text{SINR}_i$. Constraint (4) imposes that allocated power cannot be negative and the total transmission power in each cell cannot exceed a predefined threshold. Constraint (5) is to guarantee user’s rate requirement and each subcarrier is assigned to only one user.

3. New Algorithm

The above problem is a mixed optimization problem since $x_i(j)$ is binary and $p_i(j)$ is a real number. It cannot be solved with conventional method. In this paper we consider low-complexity sub-optimal method. A key observation is that minimal power is achieved when

$$G_i(j)p_i(j) = \text{SINR}_i \left( \sum_{\text{h} \in M} G_i^h(j)p_i(j) + BN_0 \right), \quad i \in S(j), j \in M \quad (6)$$

The key motivation of the proposed algorithm is that we can solve the linear equations in (6) for power allocation. $S(j)$ is required in constructing these equations, which implies subcarrier allocation should be performed before power allocation. We proposed the following steps to solve the target problem: 1) Preliminary subcarrier allocation; 2) Power allocation; 3) Adjustment of subcarrier and power in case of negative power.

3.1 Single-cell subcarrier allocation

In this step the subcarrier allocation is performed in a cell-separating manner. That is, each cell separately applies the following allocation steps.

I. Form a two-dimensional matrix, where each low corresponds to one user’s channel power gains in all subcarriers.

II. Find the maximal value in this matrix. If the corresponding user has been assigned $r_i$ subcarriers, then change the maximal value to zeros and go to next step. Otherwise assign the corresponding subcarrier to the corresponding user. Set the maximal value to zero and go to next step.

III. Go to step (II) if there exist unassigned subcarriers, otherwise end the subcarrier allocation procedure.

3.2 Multicell initial power allocaton

After finishing subcarrier allocation, the set of users using subcarrier $j$, i.e., $S(j)$, is available. Form the set of linear equations associated with subcarrier $j$, as shown in (6). Solving these linear equations result in initial power allocation.

3.3 Adjustment of resource allocation

Since negative power allocation is not possible in practice. It is required to adjust the assigned power in the case that equation (6) results in negative solution. Next we investigate the adjustment scheme based on (6). For notation simplicity, we ignore the dependency on $j$ and rewrite (6) as follows.

$$p_1 = g_{11}p_1 + g_{12}p_2 + \cdots + g_{1,L}p_L + \Delta_1$$

$$p_2 = g_{21}p_1 + g_{22}p_2 + \cdots + g_{2,L}p_L + \Delta_2$$

$$\ldots$$

$$p_L = g_{L1}p_1 + g_{L2}p_2 + \cdots + g_{LL-1,p_{L-1}} + \Delta_L$$

where

$$\Delta_i = B \cdot N_0 \cdot \text{SINR}_i \frac{G_i}{G_i^h}$$

Note $g_{ih} > 0$, $\Delta_i > 0$ $(\forall i, h, i \neq h)$. Negative power allocation will not be unique. We argue that the negative power allocation is due to the mutual interference between at least two users. For instance, two users at the cell edges who correspond to different cell but are assigned the same subcarrier [10]. We consider a specific case where there are two negative solutions at $p_1$ and $p_2$. To focus on the mutual effect of $p_1$ and $p_2$, we rewrite (7) as

$$p_1 = g_{11}p_1 + \Delta_1$$

$$p_2 = g_{21}p_1 + \Delta_2$$

where $\Delta_1 = g_{11}p_1 + \cdots + g_{1,L}p_L + \Delta_1$, $\Delta_2 = g_{21}p_1 + \cdots + g_{2,L}p_L + \Delta_2$. Eq.(8) presents two lines on the plane of $\{p_1, p_2\}$, Since $\{g_{11}, \Delta_1\}$, $\{g_{21}, \Delta_2\}$ both have positive values, the intersection point of these two lines will be in the first or third quadrant, depending on the relation between $\{g_{11}, g_{21}\}$. If $1 - g_{12}g_{21} > 0$ then the intersection point will be in the first quadrant, otherwise the intersection point will be in the third quadrant (see fig. 1). Since the values of channel gains and $\text{SINR}_i$ are fixed, adjusting these parameters is not applicable to move the intersection
point from third quadrant to first quadrant, i.e., to convert the negative solutions to positive solutions. Directly converting the negative solution to positive value is against the constraints in (3). Hence we argue that the optimal way of adjustment is to switch user’s allocated subcarrier. That is, we find out a proper un-occupied subcarrier (simply the one with maximal gain) in the serving cells of these two users with negative $p_1$ and $p_2$, then switch the allocated subcarrier to the un-occupied subcarrier.

$$p_1 = g_{12}p_2 + \Delta_1$$
$$p_2 = g_{21}p_1 + \Delta_2$$

Fig.1 Illustration of the intersection points of the two lines in (8). (a), $1 - g_{12}g_{21} > 0$, the intersection point is in the first quadrant, (b), $1 - g_{12}g_{21} > 0$, the intersection point is in the first quadrant.

Similar scheme can be applied when there are more than two users with negative solution. In this case every pair of users will be check with the criterion in fig. 1. The adjustment scheme is detailed as follows.

I. For each subcarrier, construct the equations as in (6). If eq.6 results in all positive power allocations, then the results are applicable. End the resource allocation algorithm.

II. If there exists negative power allocation, for each pair of negative power solution apply the criterion in fig.1 and perform the above subcarrier switching scheme if necessary. If there is no free subcarrier in the serving cell, turn off one user’s subcarrier.

III. Repeat step II until all users have positive power allocation, or reaching a predefined maximal number of iteration. Then go to next step.

IV. In this step, we deal with the residual negative power allocation. For the users with negative power in partial subcarriers, we directly turn off the subcarriers with negative power allocation. For the user with negative power allocation in all subcarriers, we set the allocated power to $p_j(j) = \text{SINR} \cdot B \cdot N_0 / G(j)$, which corresponds to the power allocation without ICI. This is to guarantee a basic transmission rate for this user.

It is straightforward that the proposed algorithm can easily guarantee polynomial complexity. The subcarrier allocation requires two-dimensional searching in user and subcarrier. The complexity is on the order of $O(|M| \cdot |U|)$, where $|A|$ denotes the size of set $A$.

The complexity of solving the linear equation in (6) is on the order of $O(|S(j)|^3 |M|)$ . The adjustment also requires two-dimensional searching in user and subcarrier. The complexity is on the order of $O(|S(j)|^3 |M| + |M| \cdot |U|)$.

3. Numerical Result

Three different algorithms are compared in the simulations. 1) Optimal algorithm. We relax the binary constrain of $x(j) = [0,1]$ to $x(j) = [0,1]$ and add the penalty item $\beta(x(j) - x'(j))$ with $\beta$ being a large positive number, to the objective function. The mixed optimization problem is converted to a quadratic optimization problem can solved with the optimization toolbox of MATLAB. 2) The proposed algorithm. 3) The multi-assign algorithm in [4]. In applying the proposed algorithm, the maximal iteration in the adjustment procedure is set to 10. The presented results
Simulation results are shown for an $K=7$ cell multiuser OFDMA system with cell radius $R=1000$m. The overall signal bandwidth of 5MHz divided into 32 subcarriers, so the bandwidth of each subcarrier is $B=156250$Hz. The wireless channel is modeled as a multipath channel consisting of 5 independent Rayleigh multipaths, with an exponentially decaying profile. The user locations are assumed to be uniform distributed. Assume that each user has the target rate $R=4B^*\eta_i$ and has the equal target spectral efficiency $\eta_i=2$bits/s/Hz, the number of sub-carriers assigned for any user is $r=4$.

Fig.2 shows the total transmission power when the user number in each cell vary from 2 to 8. The multi-assign algorithm increase transmission power when (3) is not satisfied. When users at the cell edge have significant mutual interference, as shown in [9], high transmission power is prospected. The proposed algorithm obtains a tradeoff between the optimal algorithm and the multi-assign algorithm.

Fig.3 shows the total system throughput for various values of user numbers. It can be observed that the optimal algorithm results in least throughput. This is because the optimal algorithm obtains the most efficient way to fulfill the rate requirement of the users. The power efficiency (in throughput per Watt) is shown in fig. 4. It can be observed that the optimal algorithm obtain best power efficiency.

Fig. 5 shows the consumed time in doing the simulations. The proposed algorithm is more efficient when the number of users is less than 6. When the number of users increases, the subcarrier adjustment is more complicate due to more significant ICI.

5. Conclusion

A low-complexity algorithm has been proposed for joint subcarrier and power allocation in OFDMA system. Simulation shows the new algorithm obtains satisfactory tradeoff between performance and complexity.

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References