Runtime Analysis of Pigeon-Inspired Optimizer Based on Average Gain Model

1st Zhang Yushan
School of Statistics and Mathematics, Guangdong University of Finance and Economics
Big Data and Educational Statistics Application Laboratory(2017WSYS001), Guangdong University of Finance and Economics
Guangzhou, China
scuthill@163.com

2nd Huang Han*
School of Software Engineering, South China University of Technology
Guangzhou, China
hhan@scut.edu.cn

3rd Hao Zhifeng
School of Mathematics and Big Data, Foshan University
Foshan, China
zfhao@fosu.edu.cn

4th Hong Zhou
Office of Academic Research, Guangzhou City Polytechnic
Guangzhou, China

***Abstract*—The pigeon-inspired optimization (PIO) algorithm is a novel swarm intelligence optimizer inspired by the homing behaviors of pigeons. Although PIO has demonstrated effectiveness and superiority in numerous fields, there are few results about the theoretical foundation of PIO. This paper employs the average gain model to estimate the upper bound for the expected first hitting time of PIO in continuous optimization. The case study and experiment result indicate that our theoretical analysis is applicable to the general case where the population size and problem size are both larger than 1, which is close to the practical situation.**

***Index Terms*—pigeon-inspired optimization (PIO), runtime analysis, average gain, first hitting time**

I. Introduction

Population-based swarm intelligence (SI) optimization algorithms, such as particle swarm optimization (PSO) [1], ant colony optimization (ACO) [2], brain storm optimization (BSO) [3], have been successfully applied to solve various complicated optimization problems over the last two decades. Through simulating the behaviors of various swarms in the nature, researchers propose an increasing number of SI optimization algorithms, these algorithms are helpful to provide practical and efficient solutions for all kinds of optimization problems.

Recently, a novel SI algorithm named pigeon-inspired optimization (PIO) algorithm, inspired by the homing behaviors of pigeons, was firstly proposed by Duan and Qiao in 2014 [4].Pigeons can easily find their homes through using three homing tools: magnetic field, the sun and landmarks. The magnetic field is employed to shape the map and adjust the homing direction in accordance with the altitude of the sun. Landmarks around pigeons guide them to fly close to the destination. In order to mimic the natural phenomena, the PIO algorithm utilizes two operators to describe the flocking behavior of homing pigeons, i.e., the map and compass operator represents effects of the magnetic field and the sun, the landmark operator describes the effects of landmarks [5], [6].

Ever since this new SI optimization algorithm was proposed, many research results have been obtained with respect to the improvements and applications of PIO. A series of comparative experiments on both some benchmarks and some practical optimization problems have indicated that the PIO has a better performance compared with other bio-inspired algorithms. Duan and Qiao [4] applied PIO to solve air robot path planning problems and found that the PIO outperformed the standard differential evolution (DE) algorithm in convergence speed and stability. To improve the performance of the power system of the unmanned aerial vehicle (UAV), a modified PIO algorithm named adjacent-disturbances and integrated-dispatching pigeon-inspired optimization (ADID-PIO) was proposed to optimize the design of parameters of a dc brushless motor [6], the comparative experimental results

This work is supported by NSFC-Guangdong Joint Fund (U1501254), National Natural Science Foundation of China (61772225, 61876207), Guangdong Natural Science Funds for Distinguished Young Scholar (2014A030306050),the Ministry of Education-China Mobile Research Funds (MCM20160206), Guangdong High-level Personnel of Special Support Program(2014TQ01X664), International Cooperation Project of Guangzhou(201807010047), Guangzhou Science and Technology Program(201804010276, 201707010227, 201707010228), Project of Innovative Academic Team of Guangzhou Educational System(1201610027).

showed that the convergence rate, the efficiency, and the stability of ADID-PIO were better than those of basic PIO, BSO, and predator-prey BSO (PPBSO) [7] in the design process of dc brushless motor. A variant of the PIO named predator-prey pigeon-inspired optimization (PPPIO) was presented to solve the uninhabited combat aerial vehicle (UCAV) three-dimension path planning problems in dynamic environment, and the comparative simulation results showed that the PPPIO algorithm was more efficient than the basic PIO, PSO and DE [5]. For more applications of the PIO and its performance comparisons with other bio-inspired algorithms, please refer to [8]–[12].

Although PIO has demonstrated effectiveness and superiority in numerous fields, the theoretical foundation of PIO is still weak [13]. Currently, the theoretical studies of PIO are mainly based on empirical and intuitive statistical results, and rigorous mathematical arguments are lacking [13]. Zhang and Duan [5] conducted a preliminary convergence analysis of PIO by treating the state of population sequence of the PIO algorithm as a *finite* Markov chain. To the best of our knowledge, this is the only literature concerning the convergence analysis of PIO found so far.

Another theoretical issue is the runtime analysis. PIO is mainly used to solve continuous optimization problems, i.e., the PIO is basically a continuous bio-inspired algorithm. Many results on the runtime analysis of continuous EAs (evolutionary algorithms) have been achieved, while similar research on PIO is still lacking. Through modeling the continuous EA as a renewal process [14], [15], Agapie analyzed the computation time of continuous EA, such as $(1+\lambda)$ ES, $(\mu+\lambda)$ ES [16].

In this paper, after a brief and clear description of PIO, we introduce the average gain model [17] to estimate the upper bound for the expected first hitting time of PIO in continuous optimization. We consider the general case where the population size and problem size are both larger than one, which is close to the practical situation. In order to validate our theoretical analysis, a case study and experiment are presented.

II. PIO and Its Stochastic Process Model

A. Introduction to PIO Algorithm

In the light of [4], the PIO uses two operators to mimic the behavior of homing pigeons, which are map and compass operator, landmark operator respectively.

(1) **Map and compass operator.** Pigeons are randomly initialized in a D-dimension search space $\mathbb{R}^D$. The total number of pigeons is N_p, $(D, N_p \in \mathbb{Z}^+)$. The position and velocity of the k-th pigeon at the t-th iteration are denoted as $\vec{x}_k(t) = (x_{k1}(t), x_{k2}(t), \cdots, x_{kD}(t))$ and $\vec{v}_k(t) = (v_{k1}(t), v_{k2}(t), \cdots, v_{kD}(t))$ respectively, where $k = 1, \cdots, N_p$,$t = 0, 1, \cdots$.

The new velocity $\vec{v}_k(t)$ and position $\vec{x}_k(t)$ at the t-th iteration are updated as follows:

$$\vec{v}_k(t) = \vec{v}_k(t-1) \cdot e^{-Rt} + r \cdot (\vec{P}_g(t-1) - \vec{x}_k(t-1)), \quad (1)$$

$$\vec{x}_k(t) = \vec{x}_k(t-1) + \vec{v}_k(t), \quad (2)$$

where $R \in (0,1)$ is the map and compass factor, $r \sim U(0,1)$ is a uniform random variable, $\vec{P}_g(t-1)$ is the best position found so far until the $(t-1)$-th iteration by the entire swarm (i.e., the global best position).

When the number of loops reaches the required number of iterations, the map and compass operator stops execution, and switches to the landmark operator.

(2) **Landmark operator.** We take maximization problem into consideration. Sort all pigeons from largest to smallest according to their fitness values. The total number of pigeons in every generation will be halved, and the pigeons in the lower half of the line sorted by fitness values are abandoned. Let $\vec{X}_C(t)$ be the center of the pigeons' positions at the t-th generation, the position updating rule for each pigeon k at iteration t can be given by:

$$N_p(t) = \text{ceil}\left(\frac{N_p(t-1)}{2}\right), \quad (3)$$

$$\vec{X}_C(t) = \frac{\sum_{j=1}^{N_p(t)} \vec{x}_j(t) \cdot fitness\left(\vec{x}_j(t)\right)}{N_p(t) \sum_{j=1}^{N_p(t)} fitness\left(\vec{x}_j(t)\right)}, \quad (4)$$

$$\vec{x}_k(t) = \vec{x}_k(t-1) + r \cdot \left(\vec{X}_C(t) - \vec{x}_k(t-1)\right), k = 1, 2, \cdots, N_p. \quad (5)$$

In the equations above, $N_p(t)$ represents the number of pigeons in the t-th generation, $N_p(0) = N_p$. The function ceil$(\cdot)$ returns the smallest integer value greater than or equal to each input value, obviously, after a quantity of iterations, $N_p(t) = \text{ceil}(\frac{Np}{2^t})$, as $\frac{Np}{2^t} \in (0,1)$, thus $N_p(t)$ will remain 1, therefore $\vec{X}_C(t)$ will be the current best position in the t-th generation. $r \sim U(0,1)$ is a uniform random variable independent of both $\vec{x}_k(t)$ and $\vec{X}_C(t)$.

The implementation procedure of PIO is described below [4], as shown in Algorithm 1.

B. Description of the Optimization Problem

Without loss of generality, we assume that the PIO algorithm analyzed in this paper is used to tackle maximization problems in continuous search space.

Definition 1 (maximization problem): Let $S = \prod_{i=1}^{D} [-a_i, a_i] \subset \mathbb{R}^D$, $a_i > 0$ be a D-dimensional continuous search space, and let $f : S \to \mathbb{R}$ be a D-dimensional function. A maximization problem is to find a global optimum $\vec{x}^* \in S$, such that $f^* \triangleq f(\vec{x}^*) = \max_{\vec{x} \in S} f(\vec{x})$.

The function $f : S \to \mathbb{R}$ is called the objective function of the maximization problem. We do not require f to be continuous, but it must be bounded. Furthermore, we only consider the unconstrained optimization.

In addition, the following properties are assumed.

(1) The subset containing global optimal solutions in S is non-empty.

Algorithm 1 Pigeon-Inspired Optimization (PIO) Algorithm

Input: N_p: number of individuals in pigeon swarm.
D: dimension of the search space.
R: the map and compass factor.
$Nc_{1\max}$: the maximum number of generations that the map and compass operation is carried out.
$Nc_{2\max}$: the maximum number of generations that the landmark operation is carried out.
Output: $\vec{P}_g$: the global best position.
1. Initialization
Set initial values for $Nc_{1\max}$, $Nc_{2\max}$, N_p, D, R.
Set initial position $\vec{x}_k$ and velocity $\vec{v}_k$ for each pigeon, $k = 1, \cdots, N_p$.
2. Map and compass operations
for $t = 1$ to $Nc_{1\max}$ **do**
  for $k = 1$ to N_p **do**
    Calculate $\vec{v}_k(t)$ and $\vec{x}_k(t)$ according to equations (1), (2);
  end for
  Evaluate $\vec{x}_k(t)$, $k = 1, \cdots, N_p$ and update $\vec{P}_g(t)$;
end for
3. Landmark operations
for $t = Nc_{1\max} + 1$ to $Nc_{2\max}$ **do**
  Rank all the pigeon individuals according to their fitness values;
  $N_p(t) = \text{ceil}\left(\frac{N_p(t-1)}{2}\right)$;
  Keep $N_p(t)$ individuals with better fitness value, and abandon the others;
  Calculate $\vec{X}_C(t)$ and update $\vec{x}_k(t)$, $k = 1, \cdots, N_p$ according to equations (4), (5);
  Evaluate $\vec{x}_k(t)$, $k = 1, \cdots, N_p$ and update $\vec{P}_g(t)$;
end for
4. Output
$\vec{P}_g(Nc_{2\max})$ is output as the global optimum.

(2) Let $S^*(\varepsilon) = \{\vec{x} \in S | f(\vec{x}) > f^* - \varepsilon\}$ be the global optimum ε-neighborhood. Each element of $S^*(\varepsilon)$ might as well be considered as an optimum of the maximization problem.

(3) For any $\varepsilon > 0$, the Lebesgue measure of $S^*(\varepsilon)$, denoted as $m(S^*(\varepsilon)) > 0$.

The first assumption describes the existence of global optima for the problem. The second assumption presents a rigorous definition of global optimum for continuous maximization problems. The third assumption shows that there always exist solutions whose objective values are distributed continuously and arbitrarily close to the global optimum, which makes the maximization problem solvable.

C. Stochastic Process Model of PIO

Our runtime analyses are based on representing the PIO algorithm as a stochastic process. In this section, we explain the notations and terminologies used throughout the rest of this article.

Definition 2 (state of pigeon swarm): The state of pigeon swarm at iteration $t (t = 0, 1, \cdots)$ is defined as $\vec{\eta}(t) = \left(\vec{x}_1(t), \cdots, \vec{x}_{N_p}(t), \vec{P}_g(t)\right)$, where $\vec{x}_1(t), \cdots, \vec{x}_{N_p}(t), \vec{P}_g(t) \in S$.

Definition 3 (state space of pigeon swarm): The set of all possible pigeon swarm states is called the state space of pigeon swarm, denoted as $\Omega = S^{N_p+1} = \left\{\vec{\eta} = \left(\vec{x}_1, \cdots, \vec{x}_{N_p}, \vec{\xi}\right) | \vec{x}_k \in S, k = 1, \cdots, N_p; \vec{\xi} \in S\right\}$.

Definition 4 (ε-global optimum state space of pigeon swarm): The ε-global optimum state space of pigeon swarm is defined as $\Omega^*(\varepsilon) = \left\{\vec{\eta} = \left(\vec{x}_1, \cdots, \vec{x}_{N_p}, \vec{\xi}\right) | \exists \vec{x}_k \in S^*(\varepsilon), k = 1, \cdots, N_p; \vec{\xi} \in S\right\}$.

Later in this article, we will discuss the first hitting time of pigeon swarm sequence to $\Omega^*(\varepsilon)$.

Definition 5 (discrete time stochastic process of PIO): The discrete time stochastic process associated with PIO algorithm is denoted as $\left\{\vec{\eta}(t) = \left(\vec{x}_1(t), \cdots, \vec{x}_{N_p}(t), \vec{P}_g(t)\right)\right\}_{t=0}^{+\infty}$, whose state space is Ω.

According to [5], $\left\{\vec{\eta}(t) = \left(\vec{x}_1(t), \cdots, \vec{x}_{N_p}(t), \vec{P}_g(t)\right)\right\}_{t=0}^{+\infty}$ is Markovian.

III. Estimation of Expected First Hitting Time Upper Bound of PIO

A. Brief Introduction to Average Gain Model

The average gain model is built on a time-discrete non-negative stochastic process represented by $\{X_t\}_{t=0}^{\infty}$. The expected one-step change $\delta_t = E(X_t - X_{t+1} | X_t, X_{t-1}, \cdots X_0), t \geq 0$ is called average gain. For any $\varepsilon > 0$, denote the first hitting time as $T_\varepsilon = \min\{t \geq 0 : X_t \leq \varepsilon\}$.

Theorem 1: [17] Suppose $\{X_t\}_{t=0}^{\infty}$ to be a Markov process with $X_t \geq 0$ for all $t \geq 0$. Let $h : [0, A] \to \mathbb{R}^+$ be a monotone increasing, integrable function. If $E(X_t - X_{t+1} | X_t) \geq h(X_t)$ when $X_t > \varepsilon > 0$, then it holds for T_ε that

$$E(T_\varepsilon | X_0) \leq 1 + \int_\varepsilon^{X_0} \frac{1}{h(x)} dx. \tag{6}$$

B. Expected First Hitting Time Upper Bound of PIO

For any pigeon swarm $\vec{\eta}(t) = \left(\vec{x}_1(t), \cdots, \vec{x}_{N_p}(t), \vec{P}_g(t)\right)$, $t = 0, 1, \cdots$, denote its fitness function as $F(\vec{\eta}(t))$, suppose $f(\cdot)$ to be the objective function of the considered maximization problem, we define

$$F(\vec{\eta}(t)) \triangleq \max\left\{f(\vec{x}_1(t)), \cdots, f\left(\vec{x}_{N_p}(t)\right), f\left(\vec{P}_g(t)\right)\right\}.$$

As $\vec{P}_g(t)$ is the best position found so far until the t-th iteration by the entire swarm (i.e., the global best position), we can let $F(\vec{\eta}(t)) = f\left(\vec{P}_g(t)\right)$, therefore $\{F(\vec{\eta}(t))\}_{t=0}^{+\infty}$ is a monotonic non-decreasing sequence.

Let $X_t = f^* - F(\vec{\eta}(t)), t = 0, 1, \cdots$, then $X_t - X_{t+1} = F(\vec{\eta}(t+1)) - F(\vec{\eta}(t))$.

Let $q_t^* = \min_{\vec{y} \in \Omega \setminus \Omega^*(\varepsilon)} P(\vec{\eta}(t+1) \in \Omega^*(\varepsilon) | \vec{\eta}(t) = \vec{y})$, $t = 0, 1, \cdots$.

Let $\alpha = \min\{F(\vec{z}) - F(\vec{y}) | \vec{z} \in \Omega^*(\varepsilon), \vec{y} \in \Omega \setminus \Omega^*(\varepsilon)\}$, as $F(\vec{z}) > f^* - \varepsilon, F(\vec{y}) \leq f^* - \varepsilon$, we get $\alpha > 0$ provided that α exists.

Now, we conclude the expected first hitting time upper bound of PIO as below:

Theorem 2: Suppose $T_\varepsilon = \min\{t \geq 0 : X_t \leq \varepsilon\}$ to be the first hitting time of PIO solving a maximization problem, given the initial state X_0, we have

$$E(T_\varepsilon | X_0) \leq 1 + \frac{1}{\alpha} \int_\varepsilon^{X_0} \frac{1}{q_t^* P(\vec{\eta}(t) \notin \Omega^*(\varepsilon))} dx. \tag{7}$$

Proof: Suppose the probability distribution function of $\vec{\eta}(t)$ to be $P_t(\vec{y})$, the conditional probability distribution function of $\vec{\eta}(t+1)$ given $\vec{\eta}(t)=\vec{y}$ to be $P_t(\vec{z}|\vec{y})$.

$$\begin{aligned}
&E(X_t - X_{t+1}|X_t) \\
&= E\left[F(\vec{\eta}(t+1)) - F(\vec{\eta}(t))\,|\vec{\eta}(t)\right] \\
&= \int_\Omega E\left[F(\vec{\eta}(t+1))\,|\vec{\eta}(t)=\vec{y}\right] dP_t(\vec{y}) - \int_\Omega F(\vec{y}) dP_t(\vec{y}) \\
&= \int_\Omega \left[\int_\Omega F(\vec{z}) dP_t(\vec{z}|\vec{y})\right] dP_t(\vec{y}) - \int_\Omega F(\vec{y})\, dP_t(\vec{y}) \\
&= \int_\Omega \left[\int_\Omega F(\vec{z}) dP_t(\vec{z}|\vec{y}) - F(\vec{y})\right] dP_t(\vec{y}) \\
&= \int_\Omega \left[\int_\Omega (F(\vec{z}) - F(\vec{y})) dP_t(\vec{z}|\vec{y})\right] dP_t(\vec{y})
\end{aligned}$$

(Noting that $\int_\Omega F(\vec{y}) dP_t(\vec{z}|\vec{y}) = F(\vec{y}) \int_\Omega dP_t(\vec{z}|\vec{y}) = F(\vec{y})$, the above last equation holds.)

$$\geq \int_{\Omega\setminus\Omega^*(\varepsilon)} \left[\int_{\Omega^*(\varepsilon)} (F(\vec{z}) - F(\vec{y})) dP_t(\vec{z}|\vec{y})\right] dP_t(\vec{y})$$

$$\begin{aligned}
&\geq \alpha \int_{\Omega\setminus\Omega^*(\varepsilon)} \left[\int_{\Omega^*(\varepsilon)} dP_t(\vec{z}|\vec{y})\right] dP_t(\vec{y}) \\
&= \alpha \int_{\Omega\setminus\Omega^*(\varepsilon)} P(\vec{\eta}(t+1) \in \Omega^*(\varepsilon)\,|\vec{\eta}(t)=\vec{y})\, dP_t(\vec{y})
\end{aligned}$$

$$\geq \alpha q_t^* \int_{\Omega\setminus\Omega^*(\varepsilon)} dP_t(\vec{y}) = \alpha q_t^* P(\vec{\eta}(t) \notin \Omega^*(\varepsilon)). \quad (8)$$

Using (6) and (8), we get (7). ■

IV. Case Study

We consider the following n-dimensional linear function as an objective function:

$$f(x_1, x_2 \cdots, x_n) = x_1 + x_2 + \cdots + x_n,$$

$$(x_1, x_2 \cdots, x_n) \in S = [0,a]\times[0,a]\times\cdots\times[0,a], a > 0. \quad (9)$$

When $(x_1, x_2 \cdots, x_n) = (a, a, \cdots, a)$, the objective function reaches its maximum value na.

To avoid complicated calculation, let the population size $N_p = 2$. Assume that the PIO starts from the initial position $(0, 0, \cdots, 0)$, then $X_0 = na$.

Theorem 3: In this case, the first hitting time T_ε satisfies:

$$E(T_\varepsilon|X_0) \leq 1 + \frac{1}{\frac{\sqrt{n}}{2\sqrt{6\pi}} e^{-\frac{3n}{2}} + \frac{n}{4}} (na - \varepsilon) \quad (10)$$

Proof: For $t = 0, 1, \cdots$, we have

$$E(X_t - X_{t+1}|X_t) = E(\eta_t|X_t) = \int_{-\infty}^{+\infty} x dF(x)$$

$$= \int_0^{+\infty} x d\left(\sqrt{\frac{6}{\pi n}} \int_{-\infty}^{x} e^{-\frac{6\left(t-\frac{n}{2}\right)^2}{n}} dt\right)$$

$$= \sqrt{\frac{6}{\pi n}} \int_0^{+\infty} x e^{-\frac{6\left(x-\frac{n}{2}\right)^2}{n}} dx$$

$$= \sqrt{\frac{6}{\pi n}} \cdot$$

$$\left(\int_0^{+\infty} \left(x - \frac{n}{2}\right) e^{-\frac{6\left(x-\frac{n}{2}\right)^2}{n}} dx + \int_0^{+\infty} \frac{n}{2} e^{-\frac{6\left(x-\frac{n}{2}\right)^2}{n}} dx\right)$$

$$= \sqrt{\frac{6}{\pi n}} \cdot$$

$$\left(-\frac{n}{12} \int_0^{+\infty} e^{-\frac{6\left(x-\frac{n}{2}\right)^2}{n}} d\left(-\frac{6\left(x-\frac{n}{2}\right)^2}{n}\right) + \frac{n}{2} \int_0^{+\infty} e^{-\frac{6\left(x-\frac{n}{2}\right)^2}{n}} dx\right)$$

$$= \sqrt{\frac{6}{\pi n}} \left(-\frac{n}{12}\left(0 - e^{-\frac{3n}{2}}\right) + \frac{n\sqrt{n}}{2\sqrt{6}} \frac{\sqrt{\pi}}{2}\right)$$

$$= \sqrt{\frac{6}{\pi n}} \left(\frac{n}{12} e^{-\frac{3n}{2}} + \frac{n\sqrt{n\pi}}{4\sqrt{6}}\right)$$

$$= \frac{\sqrt{n}}{2\sqrt{6\pi}} e^{-\frac{3n}{2}} + \frac{n}{4}$$

Hence for any $\varepsilon > 0$, according to Theorem 2, we get

$$E(T_\varepsilon|X_0) \leq 1 + \int_\varepsilon^{na} \frac{1}{\frac{\sqrt{n}}{2\sqrt{6\pi}} e^{-\frac{3n}{2}} + \frac{n}{4}} dx$$

$$= 1 + \frac{1}{\frac{\sqrt{n}}{2\sqrt{6\pi}} e^{-\frac{3n}{2}} + \frac{n}{4}} (na - \varepsilon)$$

■

Notice that

$$\lim_{n\to\infty} \left[1 + \frac{1}{\frac{\sqrt{n}}{2\sqrt{6\pi}} e^{-\frac{3n}{2}} + \frac{n}{4}} (na - \varepsilon)\right] = 1 + 4a,$$

this means the time upper bound will keep to be a constant when the problem's dimension n is sufficiently large.

V. Experiment

Theorem 3 gives the upper bound of the expected first hitting time for a simple PIO on n-dimensional linear function problem. We conduct experiment to validate the theoretical result, the experiment settings: $\varepsilon = 0.01, a = 20, n = 1, 2, \cdots, 400$. For each problem's dimension n, we conduct 300 runs of the algorithm and take the average value of the 300 first hitting time as the actual expected first hitting time. Figure 1 illustrates the comparison of the actual value and theoretical value of the expected first hitting time(EFHT).

For each $n = 1, 2, \cdots, 400$, the EFHT estimated by equation (10) is around $1 + 4a = 1 + 4 \times 20 = 81$, and the actual EFHT is around 40, which is bounded above by the former.

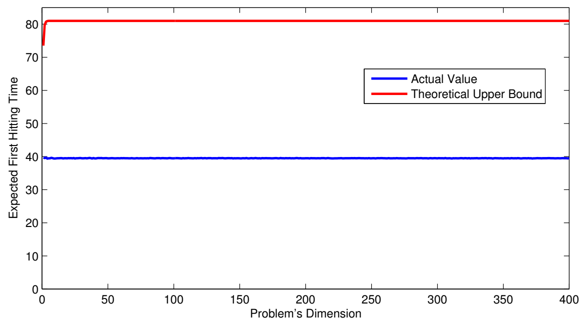

Fig. 1. Comparison of the Actual Value and Theoretical Upper Bound of the EFHT

VI. Conclusion

For the general case where both the PIO's population size and problem's dimension are larger than one, we introduce the average gain model to establish a universal formula of the expected first hitting time upper bound. In the case study, we analyze the EFHT upper bound of the PIO with 2 pigeons on the n-dimensional linear function. The numeric experiment shows that the theoretical method proposed in this paper meets actual situation. This is our preliminary effort to conduct the runtime analysis of this newly presented SI algorithm. In the future, we will use the theoretical results obtained by this paper to analyze the runtime of PIO on more problem instances.

References

[1] M. R. Bonyadi, Z. Michalewicz, "Particle swarm optimization for single objective continuous space problems: a review," Evol. Comput, vol. 25, no.1, pp.1-54, 2017.

[2] M. Dorigo, C. Blum, "Ant colony optimization theory: a survey," Theor. Comput. Sci, vol. 344, no.2-3, pp. 243-278, 2005.

[3] Y. H. Shi, "An optimization algorithm based on brainstorming process," Int. J. Swarm Intel. Res, vol. 2, no. 4, pp. 35-62, 2011.

[4] H. B. Duan, P. X. Qiao, "Pigeon-inspired optimization: a new swarm intelligence optimizer for air robot path planning," Int. J. Intell. Comput. Cybern, 2014, vol. 7, no. 1, pp. 24-37, 2014.

[5] B. Zhang, H. B. Duan, "Three-dimensional path planning for uninhabited combat aerial vehicle based on predator-prey pigeon-inspired optimization in dynamic environment," IEEE ACM T. Comput. Bi, vol. 14, no. 1, pp. 97-107, 2017.

[6] X. B. Xu, Y. B. Deng, "UAV power component–DC brushless motor design with merging adjacent-disturbances and integrated-dispatching pigeon-inspired optimization," IEEE T. Magn, vol. 54, no. 8, pp.1-7, 2018.

[7] H. B. Duan, S. T. Li, and Y. H. Shi, "Predator-Prey brain storm optimization for DC brushless motor," IEEE T. Magn, vol. 49, no. 10, pp. 5336-5340, 2013.

[8] T. Li, C. J. Zhou, B. Wang, et al, "A hybrid algorithm based on artificial bee colony and pigeon inspired optimization for 3D protein structure prediction," J. Bionanosci, vol. 12, no. 1, pp. 100-108, 2018.

[9] G. Sushnigdha, A. Joshi, "Trajectory design of re-entry vehicles using combined pigeon inspired optimization and orthogonal collocation method," IFAC-PapersOnLine, vol. 51, no. 1, pp. 656-662, 2018.

[10] L. Xin, N. Xian, "Biological object recognition approach using space variant resolution and pigeon-inspired optimization for UAV," Sci. China Technol. Sci, vol. 60, no. 10, pp.1577-1584, 2017.

[11] S. Rajendran, U. M. Sankareswaran, "A novel pigeon inspired optimization in ovarian cyst detection," Curr. Med. Imaging Rev, vol. 12, no. 1, pp. 43-49, 2016.

[12] X. J. Lei, Y. L. Ding, and F. X. Wu, "Detecting protein complexes from DPINs by density based clustering with pigeon-inspired optimization algorithm," Sci. China Inf. Sci, vol. 59, no. 7, pp. 1-14, 2016.

[13] H. B. Duan, F. Ye, "Progresses in pigeon-inspired optimization algorithms," J. B. Univ. Technol. (Nat Sci Ed), vol. 43, no. 1, pp. 1-7, 2017.

[14] A. Agapie, M. Agapie, G. Zbaganu, "Evolutionary algorithms for continuous space optimization," Int. J. Syst. Sci, vol. 44, no. 3, pp. 502-512, 2013.

[15] A. Agapie, M. Agapie, and G. Rudolph, et al, "Convergence of evolutionary algorithms on the n-dimensional continuous space," IEEE T. Cybernet, vol. 43, no. 5, pp. 1462-1472, 2013.

[16] H. G. Beyer. The Theory of Evolution Strategies. New York: Springer Press, 2001.

[17] Y. S. Zhang, H. Huang, and Z. F. Hao, et al, "First hitting time analysis of continuous evolutionary algorithms based on average gain," Clust. Comput, vol. 19, no. 3, pp. 1323-1332, 2016.